MIT JOINT PROGRAM
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Report 307
December 2016

Economic Projection with Non-homothetic Preferences: The Performance and Application of a CDE Demand System

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DECEMBER 2016

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Abstract: In computable general equilibrium modeling, whether the simulation results are consistent to a set of valid own-price and income demand elasticities that are observed empirically remains a key challenge in many modeling exercises. To address this issue, the Constant Difference of Elasticities (CDE) demand system has been adopted by some models since the 1990s. However, perhaps due to complexities of the system, the applications of CDE systems in other models are less common. Furthermore, how well the system can represent the given elasticities is rarely discussed or examined in existing literature. The study aims at bridging these gaps by revisiting calibration details of the system, exploring conditions where the calibrated elasticities of the system can better match a set of valid target elasticities, and presenting strategies to incorporate the system into GTAP8inGAMS—a global computable general equilibrium model written in GAMS and MPSGE modeling languages. It finds that the calibrated elasticities can be matched to the target ones more precisely if the corresponding sectorial expenditure shares are lower, target own-price demand elasticities are lower, and target income demand elasticities are higher. It also verifies that for the GTAP8inGAMS with a CDE system, the model responses can successfully replicate the calibrated elasticities under various price and income shocks.

1. Introduction

In Computable General Equilibrium (CGE) modeling, it has been identified that price and income elasticities of demand are crucial in determining the sectorial growth pattern and economic impacts of various policies (Hertel, 2012). This suggests that while a typical Constant Elasticity of Substitution (CES) function is still widely used in modeling final consumption (Sancho, 2009; Annabi *et al.*, 2006; Elsenburg, 2003), the property of having unitary income elasticities of demand is often considered as highly inflexible. Also, in a single-nest CES setting, after applying the Cournot's aggregation, it can be shown that the sectorial expenditure shares will fully determine the variation in own-price elasticities of demand, which is quite restrictive as well.

To capture the observed non-homothetic preferences with income elasticities of demand diverging from unity, one approach is to use the Linear Expenditure System (LES) such as the Stone-Geary preference (Geary, 1950; Stone, 1954). The LES system can be calibrated to income elasticities of demand compatible to a valid demand system. In addition, with a special multi-nest structure, the calibrated own-price elasticities of demand can be matched perfectly to any valid elasticities (Perroni and Rutherford, 1995).[1] The shortcoming of LES, however, is that due to constant marginal budget shares with respect to income, the limit property of LES is still constant-return-to-scale, and therefore the underlying income elasticities of demand will approach one as income grows.

An alternative option to model non-homotheticity is to utilize the Constant Difference of Elasticities (CDE) demand system proposed by Hanoch (1975). With implicit additivity, a N-commodity CDE system has N expansion parameters and N substitution parameters to achieve a more general functional form than the single nest CES case. The N expansion parameters make it possible to incorporate various income elasticities of demand across commodities/sectors, and the income elasticities will remain at their given levels as income changes ("commodity" and "sector" are used interchangeably in this study). On the other hand, compared to a single-nest CES setting, the N substitution parameters allow modelers to come up with a somewhat better representation for the target own-price demand elasticities.

One caveat of CDE applications, paradoxically, comes from the constancy of each income elasticity regardless of income levels. While this feature might not severely contradict empirical evidence for developed countries, existing studies have found that, for instance, income elasticities of some food items in developing countries tend to decrease as income grows (Haque, 2005; Chern *et al.*, 2003). In some cases, economic growth may turn luxury goods into necessities (Zhou *et al.*, 2012). To overcome this, with more income response parameters, Rimmer and Powell (1996) presents an implicit directly additive demand system (AIDADS) that allows income elasticities of demand to vary logistically. Nevertheless, AIDADS has a narrow range of substitution across goods, and due to theoretical and computational reasons, AIDADS applications are limited to 10 commodities/sectors (Reimer and Hertel, 2004). As a result, these applications are less common and more project-specific. In contrast, despite some limitations, the CDE system seems to be more applicable as a generic setting for modeling non-homothetic preferences.

While CGE models such as GTAP (Hertel and Tsigas, 1997), MAGNET (Woltjer and Kuiper, 2014), GTEM (ABARE/DFAT, 1995; ABARE, 1996), and ENVISAGE (van der Mensbrugghe, 2008) have been using CDE systems in modeling final consumption behaviors, perhaps due to the complexities in both calibration and implementation, other CDE applications are less common so far. More importantly, when studying the responses of CGE models with non-homothetic preferences, besides examining the implications of income elasticities of demand on future projection, the roles of own-price elasticities of demand are crucial as well, since own-price demand elasticities could also influence projections and may become even more crucial under some policy shocks. Existing literature also points out that to ensure the regularity of a well-behaved demand function, calibrating a CDE system to the target elasticities that are valid might be infeasible (Hertel, 2012; Huff *et al.*, 1997). How well the system can match those elasticities is beyond the discussion of most existing literature. One exception is Liu *et al.* (1998), which presents the differences between target and calibrated elasticities. Nevertheless, exploring sources of differences between calibrated and target elasticities is beyond the scope of that study.

Before studying how well the calibrated elasticities of a demand system can match a set of target elasticities, one needs to ensure that under a given baseline expenditure share structure, the target elasticities are valid, i.e., they are conformable to aggregation conditions and a negative semi-definite Slutsky matrix. Therefore, the demand system under consideration will only be calibrated to a set of valid target elasticities. With that in mind, the study will answer the question both analytically and numerically: given a set of valid target own-price demand elasticities, income demand elasticities and expenditure shares, under what conditions will the calibrated elasticities of a CDE system better match the target values? The findings of this

1 While Perroni and Rutherford (1995) focuses on homothetic preferences, it points out that the multi-nest strategy achieving a perfect match in own-price elasticities calibration also works for non-homothetic preferences.

study can help modelers who implement a CDE system explaining how well the target elasticities are represented in their models, and provide information for choosing an appropriate sectoral aggregation so that, if possible, at least target elasticities of interesting sectors can be better matched. Next, the author presents strategies for putting the CDE system into GTAP8inGAMS, a global CGE model written in GAMS and MPSGE using the GTAP 8 database (Rutherford, 2012). MPSGE is a subsystem of GAMS (Rutherford, 1999), and earlier it was sometimes thought that despite being a powerful tool that handles the calibration of CES functions automatically, MPSGE can only be applied to models with CES or LES utility functions (Konovalchuk, 2006; Hertel *et al.*, 1991). The study shows that the potential of MPSGE applications is far beyond what was previously perceived. The revised GTAP8inGAMS with a CDE system is tested with income and price shocks to verify the model response is consistent to the calibrated elasticities. The programs for the CDE calibration and the revised GTAP8inGAMS with a CDE system are provided in Appendix A and Appendix B, respectively, so readers can use them for verification or research purposes.

The rest of the paper is organized as follows: Section 2 briefly reviews the theories and settings of the CDE system; Section 3 presents the calibration, performance, and implementation of the CDE system; and Section 4 provides a conclusion.

2. Theoretical Background

To understand what constitutes a regular (i.e., valid) demand response, the section will briefly review the economic considerations for a regular demand system. A question that follows is: how can one evaluate the performance of a regular demand system in terms of representing the target own-price and income demand elasticities that are valid? To explore this, the section will discuss a demand system's flexibilities in own-price and income demand elasticities calibration, introduce the settings of CDE system, and finally examine the implications of CDE regularity conditions on the calibration performance of the system.

2.1 Regularity and Flexibility of a Demand System

Let us denote a cost (or expenditure) function by $C(p,u)$ where p is a N-dimensional price vector and u is the utility. For C to be considered as well-behaved, $\partial C/\partial p$, which is the Hicksian demand vector $q(p,u)$, is non-negative and homogeneous of degree zero in p, and $[\partial^2 C/\partial p_i \partial p_j]_{N\times N}$, which is the Slutsky matrix, is negative semi-definite (NSD).[2] The intuition of a NSD Slutsky matrix is: for a given utility level u, when a good becomes more expensive, it will be replaced by other cheaper alternatives; as a result, the cost increase with the new consumption bundle after the price increase will never exceed the cost increase when the bundle cannot be altered.

2 For example, see p.59 and p.933 in Mas-Colell *et al.* (1995).

The Slutsky matrix $[\partial^2 C/\partial p_i \partial p_j]_{N\times N}$, or equivalently $[\partial q/\partial p]_{N\times N}$, is symmetric and each term of the matrix is:

$$\frac{\partial q_i(p,u)}{\partial p_j} = \frac{\partial x_i(p,w)}{\partial p_j} + \frac{\partial x_i(p,w)}{\partial w} x_j(p,w) \quad (1)$$

Equation (1) is the Slutsky equation, which decomposes the impacts of a price change on the uncompensated demand $x_i(p,w)$ into the income effect and substitution effect, where w is the income (or expenditure) level. With some algebra, the Slutsky equation can also be expressed as

$$\sigma_{ij}^c = \sigma_{ij}^m + \eta_i \theta_j \quad (2)$$

where σ_{ij}^c, σ_{ij}^m, η_i, and θ_j are compensated price elasticity of commodity i, uncompensated price elasticity of i, income elasticity of i, and expenditure share of j, respectively. If both sides of (2) are divided by θ_j, one can come up with a Slutsky matrix $[\sigma_{ij}]_{N\times N}$ in the form of Allen-Uzawa elasticity of substitution (AUES) (Allen and Hicks, 1934; Uzawa, 1962) with

$$\sigma_{ij} = \sigma_{ij}^m / \theta_j + \eta_i \quad (3)$$

It can be shown that $[\sigma_{ij}]_{N\times N}$ is also symmetric, and the matrix is NSD if and only if $[\partial q/\partial p]_{N\times N}$ is NSD. Therefore, a demand system is regular means 1) the Slutsky matrix $[\sigma_{ij}]_{N\times N}$ is NSD; and 2) the Hicksian demand q is non-negative. For CGE modeling, it is necessary to ensure that the demand system is globally regular (i.e., it should remain regular everywhere in the domain of price). This is because the algorithm of the solver for finding equilibria may begin from an initial point of price and quantity combination that is far from the equilibrium levels, and in the process of solving the model, the algorithm might fail if the demand system is not globally regular, even the system is locally regular at the equilibrium points (Perroni and Rutherford, 1998).

Perroni and Rutherford (1995) defined a regular-flexible demand system as the one that is globally regular and can locally represent any valid configuration of compensated demands and the AUES matrix $[\sigma_{ij}]_{N\times N}$. Based on an inductive argument, Perroni and Rutherford proved that a demand system derived from a special version of the non-separable nstage CES function is regular-flexible. Nevertheless, in general, testing whether other demand systems are regular-flexible would need to identify the domain of a regular flexible demand system first, which is beyond the scopes of their paper and the current research. Instead of matching the entire AUES matrix under a given expenditure share structure, this study will simply focus on the ability of a demand system in matching a valid combination of ownprice elasticities, income demand elasticities, and expenditure shares. Own-price and income elasticities are usually of first-order importance in characterizing the model response, and are also the most ubiquitous data available for calibrating a demand system. In particular, this study will examine whether a global regular demand system under consideration is own-price and income flexible (i.e., if the system can be calibrated to (σ_{ii}, η_i, θ_i) consistent to any well-behaved cost function). Following this definition, for example, the demand system derived from a single-nest CES cost function is neither own-price nor income flexible. The settings of CDE and their implications on own-price and income flexibilities will be discussed below.

2.2 The CDE Demand System

Let us consider the expenditure function C with a price vector p and a Hicksian demand vector q, i.e., $c_0 = C(p_0, u) \equiv \{\min p_0 q_0 : f(q_0) \geq u\}$ where the subscript 0 denotes the benchmark condition. If the function is normalized by c_0, it becomes $C(p_0/c_0, u) \equiv 1$. With this normalization, Hanoch (1975) proposes the expenditure function of a CDE demand system as follows:

$$C\left(\frac{p}{c_0}, u\right) = \sum_i \beta_i u^{e_i(1-\alpha_i)} \left(\frac{p_i}{c_0}\right)^{1-\alpha_i} \equiv 1 \qquad (4)$$

where α_i and e_i are the substitution parameter and expansion parameter, respectively. In this setting, the utility u is only implicitly defined, and in general there is no reduced form representation for u. The Hicksian demand for commodity i based on this setting is:

$$q_i = \frac{\left[\beta_i u^{e_i(1-\alpha_i)}(1-\alpha_i)\left(\frac{p_i}{c_0}\right)^{-\alpha_i}\right]}{\sum_i \beta_i u^{e_i(1-\alpha_i)}(1-\alpha_i)\left(\frac{p_i}{c_0}\right)^{1-\alpha_i}} \qquad (5)$$

For the CDE system, the substitution elasticity σ_{ij} in AUES form is presented in Equation (6), where the expenditure share is denoted by θ_i, and $\delta_{ij} = 1$ if $i = j$, otherwise $\delta_{ij} = 0$. The income elasticity of demand η_i is presented in Equation (7):

$$\sigma_{ij} = \alpha_i + \alpha_j - \sum_k \theta_k \alpha_k - \frac{\delta_{ij}\alpha_i}{\theta_i} \qquad (6)$$

$$\eta_i = \left(\sum_k \theta_k e_k\right)^{-1}\left[e_i(1-\alpha_i) + \sum_k \theta_k e_k \alpha_k\right] + \left(\alpha_i - \sum_k \theta_k \alpha_k\right) \qquad (7)$$

The following aggregation conditions hold: the Cournot's aggregation $\sum_i \theta_i \sigma_{ij} = 0$ and the Engel's aggregation $\sum_i \theta_i \eta_i = 1$. Note that for each off-diagonal term, $\sigma_{ij} - \sigma_{ik} = \alpha_j - \alpha_k$ is invariant to i although σ_{ij} may vary, and therefore the system has a constant difference of (substitution) elasticities. The regularity condition for the system presented in Hanoch (1975) includes: $\beta_i \geq 0$; $e_i \geq 0$; $0 < \alpha_i < 1$ or $\alpha_i \geq 1 \forall i$ and $\alpha_I > 1$ for some $I \in i$. It is worth noting that with the regularity condition, each own-price elasticity of demand σ_{ii}^c is always negative. This is because from Equation (6) and $\sigma_{ii}^c = \sigma_{ii}\theta_i$, we have

$$\sigma_{ii}^c = -\alpha_i(1-\theta_i)^2 - \theta_i \sum_{k|k\neq i} \theta_k \alpha \qquad (8)$$

For a given vector of θ_i, the requirement that all α_is should lie on the same side of one imposes a constraint in choosing the vector of α_i such that σ_{ii}^c can match the target own-price demand elasticity. For instance, some sectors may have a very small expenditure share ($\theta_i \rightarrow 0$) and so for those sectors $\sigma_{ii}^c \rightarrow -\alpha_i$. However, for those sectors, if some target own-price elasticities do not lie on the same side of one, it would be impossible to match every single σ_{ii}^c with the target elasticity value no matter what regulatory condition on α_i is chosen. Therefore, the CDE system is not own-price flexible. Further, the requirement of $e_i \geq 0$ also suggests that some compromise has to be made in calibrating income elasticities of demand.

3. Calibration, Performance, and Implementation

Two CDE calibration approaches have been presented. The first is the three-step procedure documented in Hertel *et al.* (1991) and Huff *et al.* (1997). In this approach, own-price demand elasticities are calibrated to target levels first. Taking parameters determined in the first step as given, income elasticities of demand are calibrated to target levels next, and scale parameters of the system are specified last. The second method is the maximum entropy approach presented by Surry (1997) and Liu *et al.* (1998). Rather than calibrating the system sequentially, the idea of this approach is to calibrating all parameters simultaneously by maximizing an objective function that considers matching both own-price and income elasticities of demand. This study will take the first approach as an example and explore under what circumstances the calibrated elasticities can better match the target elasticities, the section will examine the performance of CDE calibration both analytically and numerically. It will also demonstrate how to put the CDE system into GTAP8inGAMS and verify the model response is consistent to the calibrated elasticities.

3.1 Calibration

Step 1: ***Calibrating the own-price elasticity of demand*** σ_{ii}^{c}**.** Let us denote the target own-price elasticity of demand by σ_{ii}^{ct}. The purpose of this step is to choose α_i so that the "distance" between the two vectors $[\sigma_{ii}^{c}]$ and $[\sigma_{ii}^{ct}]$ is minimized.[3] In this study, the following function is considered for the minimization problem:

$$\min_{\alpha_i} \sum_i \omega_i (\sigma_{ii}^{c} - \sigma_{ii}^{ct})^2 \ \ s.t. \alpha_i \in (0,1) \tag{9}$$
$$or\ \alpha_i \geq 1\ \forall\ i\ and\ \alpha_I > 1\ for\ some\ I \in i$$

where $\omega_i = \theta_i$. The study will compare the performances of different settings in matching the target own-price demand elasticities.

Step 2: ***Calibrating the income elasticity of demand.*** Let us denote the target income elasticity of demand by η_i^t (η_i^t must satisfy the Engel's aggregation). Given α_i determined in the previous step, by choosing e_i, the goal is to calibrate η_i to η_i^t if possible. Similar to the idea of Step 1, the following problem is solved:

$$\min_{e_i|\alpha_i} \sum_i \omega_i (\eta_i - \eta_i^t)^2 \ \ s.t. \sum_i \theta_i \eta_i = 1 \tag{10}$$
$$(\eta_i - 1)(\eta_i^t - 1) > 0\ for\ all\ i$$

The condition $\sum_i \theta_i \eta_i = 1$ is to ensure the calibrated elasticities satisfy the Engel's aggregation, and following Huff *et al.* (1997), the second condition is to ensure the calibrated elasticities lie on the same side of one as the target values.

Step 3: ***Calibrating the scale coefficients holding the utility level equals one.*** With the calibrated α_i and e_i, and the normalization $u=1$, $p_{0i}=1$, and $q_{0i}=\theta_i$ (since $c_0 = \sum_i p_{0i} q_{0i} = 1$), the N scale parameters β_i can be solved by using (4) and (5):

$$\beta_i = \frac{q_{0i}}{1-\alpha_i} / \sum_k \frac{q_{0k}}{1-\alpha_k} \tag{11}$$

Because the calibration is done sequentially, how well the income elasticities of demand can be matched to target levels is also affected by the calibration of own-price demand elasticities. In Appendix A, the study provides the program for the three-step strategy. The program is written in GAMS, and each minimization problem in the program is formulated as a nonlinear programming (NLP) problem.

3 Without explicitly considering the distance metric, the objective function of this problem considered in Huff *et al.* (1997) is $f(\sigma_{ii}^{c}) = \sum_i \sigma_{ii}^{c} [ln(\sigma_{ii}^{c}/\sigma_{ii}^{ct}) - 1]$.

3.2 Performance

Before putting the system into a CGE model, two interesting questions are: under what circumstances does the calibration become more accurate, and how well are the target elasticities represented? The following analysis will answer these questions.

Proposition 3.2.1:

The lower the expenditure share, the higher the influence of own-sector substitution parameter in determining the calibrated own-price elasticity of demand. On the other hand, the higher the expenditure share, the higher the influence of other sectors' substitution parameters in determining the calibrated elasticity.

Proof:

Since $\sigma_{ii}^{c} = -\alpha_i(1-\theta_i)^2 - \theta_i \sum_{k|k\neq i} \theta_k \alpha_k$, with a lower θ_i ($\theta_i \in (0,1)$), σ_{ii}^{c} depends more on the own-sector substitution parameter α_i, rather than the weighted average of other sectors' substitution parameters $\sum_{k|k\neq i} \theta_k \alpha_k$. In the extreme case with $\theta_i \to 0$, if the regularity condition is not violated, σ_{ii}^{c} can be matched to the target level σ_{ii}^{ct} by simply setting $\alpha_i = -\sigma_{ii}^{ct}$ since $\lim_{\theta_i \to 0} \sigma_{ii}^{c} = -\alpha_i$. On the other hand, with a higher θ_i, σ_{ii}^{c} depends more on the weighted average of other sectors' substitution parameters $\sum_{k|k\neq i} \theta_k \alpha_k$ rather than the own-sector substitution parameter α_i. In the extreme case with $\theta_i \to 1$, α_i has no control over σ_{ii}^{c} since $\lim_{\theta_i \to 0} \sigma_{ii}^{c} = \sum_{k|k\neq i} \theta_k \alpha_k$.

Since the compensated own-price elasticities of demand presented in GTAP 8 are between -1 and 0, based on discussions above, considering the regularity condition with $\alpha_i \in (0,1)$ produces more accurate calibration results for sectors with smaller expenditure shares. With a higher sectorial resolution, more commodities/sectors will have smaller expenditure shares, and thus having $\alpha_i \in (0,1)$ will make it possible for producing a better match between calibrated and target levels for each individual sector.

Proposition 3.2.2:

When $\alpha_i \in (0,1)$, *calibrating the income elasticity of demand to a higher level is less likely to violate* $e_i \geq 0$, *which is part of the regularity condition. On the other hand, when* $\alpha_i \geq 1\ \forall\ i$ *and* $\alpha_I > 1$ *for some* $I \in i$, *calibrating the elasticity to a lower level is less likely to violate* $e_i \geq 0$.

Proof:

From Equation (7),
$e_i = \{\sum_k \theta_k e_k [\eta_i - (\alpha_i - \sum_k \theta_k \alpha_k)] - \sum_k \theta_k e_k \alpha_k\} / (1-\alpha_i)$.
When $\alpha_i \in (0,1)$, a positive numerator for the

equation above is needed to ensure $e_i \geq 0$. Therefore, other things being equal, with a higher calibrated income elasticity of demand η_i, the numerator is less likely to become negative. Similarly, for $\alpha_i \geq 1 \ \forall \ i$ and $\alpha_I > 1$, a lower η_i is less likely to violate $e_i \geq 0$.

If one considers $\alpha_i \in (0,1)$, the second proposition suggests that matching the target income elasticities for the demand of agricultural products might be trickier, since in general these products tend to have lower income elasticity values; as a result, the calibrated income demand elasticities for these products might end up with levels higher than the target numbers. Nevertheless, the values of α_i determined in Step 1 of the calibration procedure may also affect how well the target income elasticities of demand are met, as will be explored in the next proposition.

Proposition 3.2.3:

When $\alpha_i \in (0,1)$, calibrating the income elasticity of demand to a target level is less likely to violate $e_i \geq 0$ with a smaller α_i. On the other hand, when $\alpha_i \geq 1 \ \forall \ i$ and $\alpha_I > 1$ for some $I \in i$, calibrating the elasticity to the target level is less likely to violate $e_i \geq 0$ with a larger α_i.

Proof:

This can be verified by $e_i = \{\sum_k \theta_k e_k \ \ [\eta_i - (\alpha_i - \sum_k \theta_k \alpha_k)] - \sum_k \theta_k e_k \alpha_k\} / (1 - \alpha_i)$.

Continuing our previous example for commodities with low income elasticities of demand and with $\alpha_i \in (0,1)$, while Proposition 3.2.2 says that for given values of α_i, it is harder to calibrate the income elasticity of demand to a lower value, Proposition 3.2.3 suggests that if the calibrated α_i is small enough, it is still possible to calibrate the income elasticity of demand to a lower level.

Proposition 3.2.4:

Commodities with substitution parameters α_i close to one will have similar calibrated income elasticities of demand.

Proof:

From Equation (7),
$\lim_{\alpha_i \to 1} \eta_i = \sum_k \theta_k e_k \alpha_k / \sum_k \theta_k e_k + 1 - \sum_k \theta_k \alpha_k = \lim_{\alpha_j \to 1} \eta_j$.

Proposition 3.2.4 shows that the calibrated α_i may work against the calibration for income elasticities of demand. For instance, if there are two commodities with α_i and α_j both approaching unity, according to the proposition, the calibrated income elasticities of demand η_i and η_j will be very close to each other, even if their target values η_i^t and η_j^t are quite different.

To show how different sectorial aggregation levels could affect the accuracy of elasticity calibration, the study considers several different aggregation levels (**Table 1**).[4] For demonstration purpose, all GTAP regions are combined into a single region using the aggregation routine of GTAP8inGAMS. In particular, wherever needed, target elasticities are aggregated based on expenditure shares. It is worth noting that the 10-sector income demand elasticity estimates based on an implicit directly additive demand system (AIDADS) were mapped to and used as the target income demand elasticities of the original GTAP database, and following Zeitsch *et al.* (1991), income demand elasticities are then used to compute the own-price

4 For all settings, there is a single aggregated region and 2 aggregated primary factors: labor and capital.

Table 1. Settings for calibration exercises with various sectorial aggregation levels.

Aggregation Level	# of Sectors	Settings
1r3s2f	**3**	Combine GTAP sectors 1–14 (g01–g14) & 22–26 (g22–g26) into s01 (agriculture); g15–g21 & g27–g46 into s02 (manufacturing); and g47–g57 into s03 (service).
1r4s2f	**4**	Similar to **1r3s2f**, except the service sector is disaggregated into a trade and transport sector (g47–g51) and a service sector (g52–g57).
1r5s2f	**5**	Combine g01–g17 into s01; g18–g27 into s02; …; g48–g57 into s05.
1r8s2f	**8**	Combine g01–g15 into s01; g16–g21 into s02; …; g52–g57 into s08.
1r16s2f	**16**	Combine g01–g12 into s01; g13–g15 into s02; g16–g18 into s03; …; g55–g57 into s16.
1r29s2f	**29**	Combine g01–g02 into s01; g03–g04 into s02; …; g55–g56 into s28; g57 becomes s29.
1r57s2f	**57**	Keep the original GTAP sectors (g01–g57).

demand elasticities of the database, as documented in Hertel *et al.* (2014).

To assess the calibration performance for each type of elasticity, in addition to a one-by-one comparison between calibrated and target numbers for each commodity, it is informative to have an index for measuring how far the point of calibrated elasticities is from the point of target elasticities as follows:

$$d = \sqrt{\sum_{i=1}^{N} \omega_i \cdot (x_i - x_i^t)^2} \qquad (12)$$

Depending on the type of elasticity evaluated, x_i in Equation (12) could be either the own-price elasticity of demand σ_{ii}^c or the income elasticity of demand η_i, while the superscript x_i^t denotes target value and $\omega_i = \theta_i$.

When the 57 GTAP sectors are aggregated into a 3-sector setting, even the smallest sectorial expenditure share, denoted by θ_{min}, approximates 12%, and with this setting the largest share θ_{max} exceeds 63%. As the sectorial resolution increases, the difference between θ_{max} and θ_{min} is reduced. In the most disaggregated case where all 57 GTAP sectors are kept, θ_{max} is slightly above 17% and θ_{min} is only 0.0002% (**Table 2**). Per compensated own-price demand elasticity targets, the range between the largest one σ_{max}^{ct} and the smallest one σ_{min}^{ct} increases as the sectorial resolution gets higher, since more disaggregated setting means extreme values are more likely to appear. In general, σ_{max}^{ct} becomes larger ($|\sigma_{max}^{ct}|$ becomes smaller, i.e., less elastic) and σ_{min}^{ct} becomes smaller ($|\sigma_{min}^{ct}|$ becomes larger, i.e., more elastic) as the sectorial resolution increases. The same story applies to the income demand elasticity targets—with more disaggregated sectors, the range between η_{max}^{ct} and η_{min}^{ct} increases as η_{min}^{ct} becomes smaller (less elastic) and/or η_{max}^{ct} becomes larger (more elastic).When trying to calibrate the CDE system to the target own-price and income demand elasticities, it is important to verify if the target own-price demand elasticities are compatible to an AUES matrix that is NSD. For instance, with the 3-sector setting, based on the Cournot aggregation, the three off-diagonal terms of the AUES matrix are fully determined once the own-price demand elasticities in AUES form (i.e., the diagonal terms of the matrix) are given, and hence the whole AUES matrix is identified. However, this will not be a valid AUES matrix since it is not NSD, which means the target own-price demand elasticities under the three-sector setting are invalid, and one cannot claim the CDE system is not flexible based on this setting. On the other hand, in the 4-sector, 5-sector, 8-sector, and 16-sector settings, it can be shown that under each setting, the target own-price demand elasticities are compatible to an AUES matrix that is NSD, and therefore the target elasticities are valid. More specifically, if one denote the number of sectors/commodities by n, there will be $n \cdot (n-1)/2 - n$ free variables that are off-diagonal terms in an AUES matrix. Therefore, once the diagonal terms (compensated own-price demand elasticities in AUES form) are given, one can use random number generators to assign values for those off-diagonal terms (cross-price demand elasticities in AUES form), and then choose the combination that yields a NSD AUES matrix. The MATLAB subroutine for doing this job is presented in Appendix C.

Since with various sectorial aggregation levels, own-price demand elasticity targets are all between 0 and 1, to calibrate the CDE system, similar to Huff *et al.* (1997), the study chooses $\alpha_i \in (0,1)$, a setting that produces a more accurate own-price demand elasticity calibration when the sectorial resolution becomes higher or the sectors under consideration have smaller expenditure shares, based on Proposition 3.2.1. The study finds that in the 4-sector, 5-sector, 8-sector, and 16-sector settings, the calibrated own-price demand elasticities cannot match their target levels since the distance measure d_σ for each of these settings is nonzero. Nevertheless, in general, d_σ gets smaller as the sectoral resolution increases (Table 2). Indeed, if one moves further to the 29-sector or 57-sector settings, a perfect match between the calibrated own-price demand elasticities and their target levels is achieved since $d_\sigma = 0$ in both cases. Also, as sectorial shares are smaller, the calibrated own-price demand elasticity σ_{ii}^c will be closer to $-\alpha_i$ (Appendix D). These findings can also be explained by Proposition 3.2.1.

The results also show that the calibrated income demand elasticities fail to match their target levels in the 4-sector, 5-sector, 8-sector, and 16-sector settings (Table 2). Taking the first sector (agricultural sector) in the 4-sector setting for instance, the target income demand elasticity is 0.7300, while the calibrated level is 0.8442 (Appendix D), which is almost 16% off. As discussed earlier, under the sequential calibration strategy considered in this study, calibrated income demand elasticities are determined after the calibrated own-price demand elasticities. Therefore, given a set of substitution parameter $\{\alpha_i | \alpha_i \in (0,1)\}$ that specifies the own-price demand elasticities, from the perspective of income elasticity calibration, it would be trickier to target a lower income elasticity level such as one for an agricultural commodity (Proposition 3.2.2), and this explains the why the exact match between the calibrated and target income demand elasticities cannot be achieved in the 4-sector setting.

Also, under the 5-sector setting, the calibrated income demand elasticity of the first sector can match its target level perfectly, and yet that level (0.5504) is even lower than the target demand elasticity of the first sector (0.7300) under the 4-sector setting. Note that under the 5-sector setting, the substitution parameter (α_i) of the first sector (0.2552) is much smaller than that of the 4-sector setting (0.4659)—a smaller α_i would make it easier for the income demand elasticity calibration of commodity i (Proposition 3.2.3). Another finding is when there are multiple sectors with their own α_i close to 1, the calibrated income demand elasticities will converge to the same level, despite the fact that the target elasticity levels are different (Appendix D). Proposition 3.2.4 provides the explanation to this observation. Finally, with 29-sector and 57-sector settings, while the targets for income demand elasticities tend to be more extreme, a perfect match between calibrated and target levels is achieved with the help of smaller α_i (Proposition 3.2.3).

3.3 Implementation

With the calibrated parameters, the study demonstrates how to put the CDE system into the multi-region and multi-sector CGE model of GTAP8inGAMS. The original CGE model is constructed based on CES technologies for both production and final consumption. It includes a series of mixed complementary problems (MCP) (Mathiesen, 1985; Rutherford, 1995; Ferris and Peng, 1997) written in MPSGE, a subsystem of GAMS (Rutherford, 1999). To implement the CDE system, the CES expenditure function is dropped, and by declaring auxiliary variables and equations in MPSGE to formulate relevant MCP, three sets of conditions below are incorporated into the revised model:

- **The equation for total expenditure.** The total expenditure c for purchasing one unit of utility (Equation (4)) is added into the model to form a MCP with a complementarity variable c. Note that in Equation (4), c is only implicitly defined. The purpose of this problem is to determine c jointly with other conditions. As previously mentioned, in the benchmark, both the utility level and price indices of commodities are normalized to unity.

Table 2. Summary statistics, calibration performance, and validity of the AUES matrix.

Setting	**1r3s2f**	**1r4s2f**	**1r5s2f**	**1r8s2f**	**1r16s2f**	**1r29s2f**	**1r57s2f**
Number of sectors	3	4	5	8	16	29	57
Target values summary statistics							
Sectorial expenditure share							
θ_{max}	63.4297%	39.5532%	46.2424%	39.5532%	26.4814%	20.4395%	17.1860%
θ_{min}	11.7793%	11.7793%	3.4324%	2.2792%	0.0924%	0.0167%	0.0002%
Own-price demand elasticity							
σ^{ct}_{max}	-0.4294	-0.4294	-0.2056	-0.1942	-0.1669	-0.0936	-0.0711
σ^{ct}_{min}	-0.7658	-0.7800	-0.7608	-0.7800	-0.7974	-0.7957	-0.8095
σ^{ct}_{avg}	-0.6201	-0.6542	-0.5807	-0.6022	-0.6093	-0.5331	-0.5294
σ^{ct}_{std}	0.1410	0.1363	0.2064	0.1813	0.1634	0.2269	0.2220
Income demand elasticity							
η^{ct}_{max}	1.0502	1.0543	1.0513	1.0543	1.0987	1.0916	1.1190
η^{ct}_{min}	0.7300	0.7300	0.5504	0.5387	0.4874	0.3382	0.2704
η^{ct}_{avg}	0.9267	0.9569	0.8947	0.9181	0.9457	0.8851	0.8970
η^{ct}_{std}	0.1406	0.1326	0.1920	0.1708	0.1547	0.2344	0.2272
Calibration results with $\alpha\_i \in (0,1)$							
Match each σ_{ii}?	✘	✘	✘	✘	✘	✔	✔
d_σ	0.3470	0.1313	0.1856	0.1427	0.0405	0.0000	0.0000
Match each η_{ii}?	✘	✘	✘	✘	✘	✔	✔
d_η	0.2363	0.1021	0.0041	0.0081	0.0141	0.0000	0.0000
Validity of the AUES matrix							
Compatible to a NSD AUES?	✘	✔	✔	✔	✔	✔	✔

- **The equation for final demand.** The equation for final demand (Equation (5)) is coupled with its complementarity variable, the activity level of final demand, to form a MCP. The problem is incorporated into the model to solve for the final demand of each commodity.
- **The zero profit condition for utility.** Let us denote the marginal cost and marginal revenue of utility (i.e., price of utility) by mcu and pu, respectively.[5] The zero profit condition of utility and the activity level of utility compose another MCP:

$$mcu \geq pu; u \geq 0;\ (mcu - pu) \cdot u = 0; \quad (13)$$

$$mcu = \frac{c \sum_i \beta_i e_i (1-\alpha_i) u^{e_i(1-\alpha_i)-1} \left(\frac{p_i}{c}\right)^{1-\alpha_i}}{\sum_i \beta_i (1-\alpha_i) u^{e_i(1-\alpha_i)} \left(\frac{p_i}{c}\right)^{1-\alpha_i}}$$

Condition (13) states that in equilibrium, if the supply of utility u is positive, the marginal cost of utility mcu must equal the marginal revenue pu, and if mcu is higher than pu in equilibrium, u must be zero.

With the commodity price being a complementarity variable, the market clearing condition of each commodity is also formulated as a MCP by comparing the commodity supply (determined by its zero profit condition) with the final demand shown above plus the intermediate demand derived from a CES cost function as the original GTAP8inGAMS. Similarly, with the price of utility being the complementarity variable, the supply of utility combined with the demand for utility ($income/pu$) make up the MCP for the market clearing condition of utility. The model code is provided in Appendix B, and interested readers may refer to Rutherford (1999) and Markusen (2013) for details of MPSGE.

For demonstration purposes, the study considers a setting with the aggregation level of two regions, four sectors, and one primary factor, and denotes this setting by "2r4s1f." The two regions are USA and the rest of the world (ROW); four sectors are agriculture (agri), manufacturing (man), trade and transport (tran), and service (serv), following the sectorial classification for the setting "1r4s2f" presented in Table 1; and the only one primary factor is the aggregation of all primary factors of GTAP8. As before, prior to conduct and evaluate the CDE calibration, one needs to check if the target elasticities under this setting (2r4s1f) are consistent to an AUES matrix that is NSD, and it can be shown that this is indeed the case (the NSD AUES matrix can be found numerically based on the subroutine presented in Appendix C). With the 2-region and 4-sector setting, **Table 3** presents the calibration performance for the CDE system.

Let us parameterize the revised CGE model of GTAP8inGAMS, based on calibrated parameters in Table 3. In the model, the aggregated primary factor along with the choice of the numeraire, which is the price for the aggregated primary factor, facilitate the identification of income effect. Now, to verify whether the CDE system is correctly implemented, the study will test if the outputs of the CGE model are consistent to the underlying calibrated elasticities under given price or income shocks. For example, with the shock on the price of agricultural product in the U.S., the first exercise changes the cost of final consumption for agricultural product in the U.S. ex-

5 mcu in Condition (13) can be derived by taking the total derivative of Equation (4) with respect to u and c at a given commodity price vector.

Table 3. Performance of the CDE Calibration under the setting "2r4s1f"

	θ_i	α_i	e_i	σ_{ii}^{ct}	σ_{ii}^{cc}	η_i^t	η_i^c
Region: USA							
agri	0.04909	0.85705	2.00000	-0.67034	-0.82165	0.81292	0.99981
man	0.18381	0.99999	0.00000	-0.82044	-0.81489	0.99514	1.00000
tran	0.20250	0.99999	0.00000	-0.85294	-0.79607	1.01152	1.00000
serv	0.56460	0.99999	3.37350	-0.85273	-0.43143	1.01372	1.00002
Distance					0.31937		0.04303
Region: ROW							
agri	0.14694	0.39172	0.18712	-0.39520	-0.40556	0.71822	0.71822
man	0.27510	0.87997	0.18541	-0.62097	-0.63723	1.00104	1.00104
tran	0.25415	0.99999	0.00000	-0.70506	-0.71473	1.05431	1.07113
serv	0.32380	0.99999	1.13413	-0.72614	-0.63656	1.08436	1.07116
Distance					0.05206		0.01133

ogenously to create the considered price shock.[6] The goal is to calculate the uncompensated (Marshallian) average own-price elasticity for the demand of agricultural product based on the model response, and see if the realized elasticity from the model output is consistent to the calibrated level.

It is worth noting that while the target own-price elasticity for the demand of agricultural product is $\sigma_{ii}^{ct}=-0.6703$, the calibrated own-price demand elasticity is $\sigma_{ii}^{cc}=-0.8217$, which again is evidence that the CDE system is not own-price flexible (Table 3). Besides, since with a nontrivial price shock imposed on the CGE model, it is more convenient to derive a "realized" uncompensated average demand elasticity based on the model's output, for comparison purposes, the study will also convert the calibrated own-price demand elasticity σ_{ii}^{cc}, which is a compensated point elasticity, into an uncompensated average demand elasticity with the same price shock so one can easily compare the realized level to the calibrated one.

The calibrated uncompensated own-price demand elasticity, $\sigma_{ii}^{m}=-0.8707$ (a point elasticity), can be derived from σ_{ii}^{c}, η_i, and θ_i based on the Slutsky equation presented in Equation (2). Let us consider the quantity index $\tilde{q}_i=q_i/\theta_i$ with the benchmark level $\tilde{q}_{0i}=1$ since $q_{oi}=\theta_i$ (see Step 3 in Section 3.1). Because the percentage change in $\tilde{q}_i$ is equivalent to the percentage change in q_i, $\tilde{q}_i$ can replace q_i in deriving the average uncompensated (Marshallian) demand elasticity —with both price and quantity indices normalized to unity, σ_{ii}^{ma} can be expressed as:

$$\sigma_{ii}^{ma} = \frac{p_i^{\sigma_{ii}^{m}}-1}{p_i-1}\cdot\frac{p_i+1}{p_i^{\sigma_{ii}^{m}}+1}; \qquad (14)$$

p_i is the after-shock price level

When various price shocks of agricultural product are in place, the values for σ_{ii}^{ma} (the calibrated average Marshallian demand elasticity) and the realized average elasticity levels σ_{ii}^{mar} (derived from the model output) are both presented in **Figure 1**. Note that with the exogenous price shocks in agricultural product, in the new equilibrium, one may also observe changes in prices of other commodities relative to their pre-shock levels, and this will in turn affect the equilibrium food consumption level due to the existence of cross-price elasticities of food demand. The exogenous price shock may also induce an income effect as reflected by the change in total (final) expenditure level. Therefore, to calculate σ_{ii}^{mar}, the consumption

6 For instance, in the revised CGE model of GTAP8inGAMS, a 10% increase in the price of agricultural product is achieved by multiplying both vdfm("agri", c, "usa") and vifm ("agri", c, "usa") by 1.1.

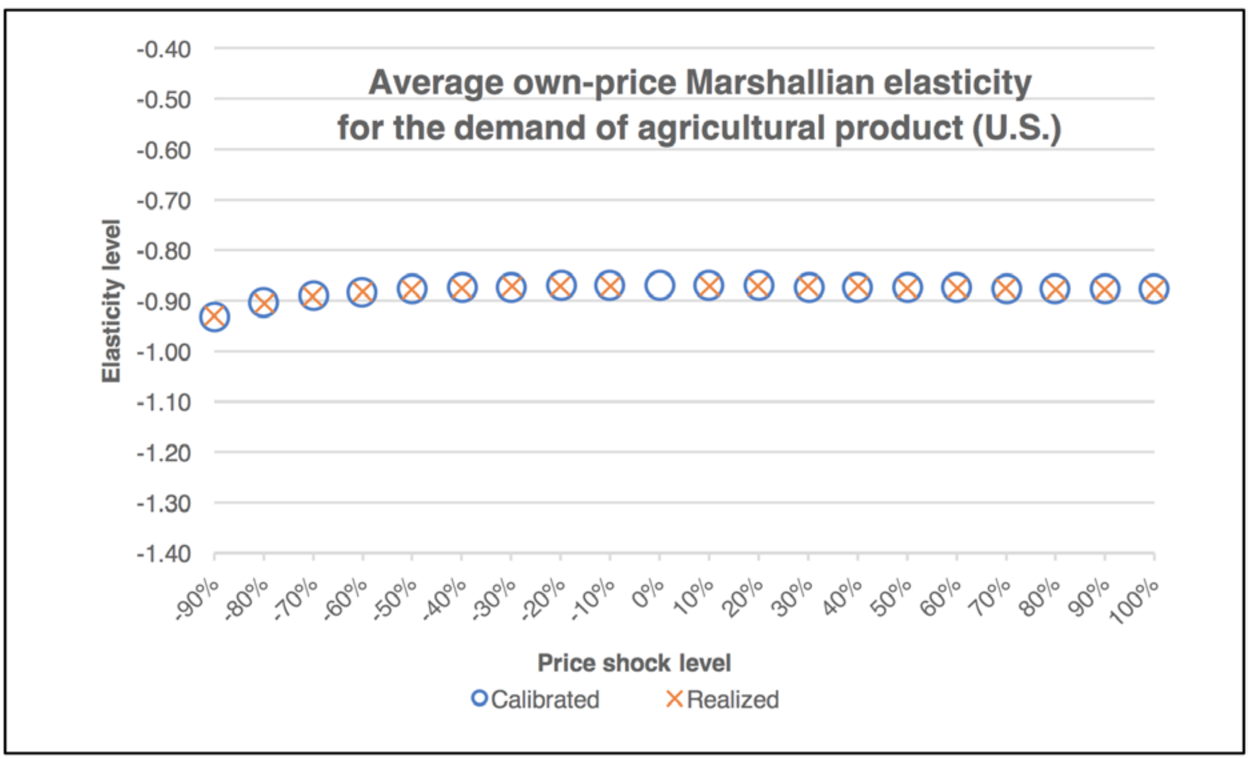

Figure 1. Average own-price elasticity for the demand of agricultural product in the U.S.

index $\tilde{q}_i$ is adjusted such that it is net of the cross-price and income effects. The result in Figure 1 shows that, as expected, the larger the price shock, the more the average elasticity deviates from the point elasticity σ_{ii}^{m}, which is the calibrated level without any price shock in the figure. Figure 1 also verifies that the uncompensated average demand elasticity σ_{ii}^{mar} calculated from the model output replicates its calibrated counterpart σ_{ii}^{ma}.

In the following exercise, the study examines the model response under various income shocks in the U.S. The shocks are created by changing the quantity of the aggregated primary factor of the U.S., which is just the real GDP level of the U.S. Since GDP is not only spent on private consumption, to calculate the income elasticities of various commodities based on the model response, instead of using the percentage change in GDP as the denominator of the elasticity, one needs to use the percentage change in the portion of income dedicated to private consumption, or equivalently, the percentage change in total expenditure on private consumption. Following the same logic as Equation (14), the average income demand elasticity can be written as:

$$\eta_i^a = \frac{c^{\eta_i}-1}{c-1} \cdot \frac{c+1}{c^{\eta_i}+1}; \qquad (15)$$

c *is the after-shock income level*

Under various levels of income shock, Equation (15) is used to convert the calibrated point elasticity into the calibrated average elasticity, which serves as the benchmark for the comparison between the realized average elasticity from model outputs and the calibrated level the model is given. Finally, as the previous example, the new equilibrium with an income shock,

will generally accompany changes in price levels of various commodities. This means that the resulting consumption levels will be contaminated by changes in prices, although these changes are usually small. The study accounts for this price effect and removes it from the consumption levels, and then for each commodity, uses the percentage change of the adjusted consumption level as the numerator of the income elasticity. **Figure 2** demonstrates that for the final consumption of agricultural product, the realized average income demand elasticity levels, as expected, replicate their calibrated counterparts. The two exercises presented here can be extended to other sectors and regions. For instance, with this 2-region and 4-sector setting, most of the calibrated income demand elasticities are close to one. The only exception is the income demand elasticity for the agricultural product in the rest of the world, $\eta_i^c = 0.7182$ (Table 3). For this elasticity, the calibrated and the realized numbers are matched as well (**Figure 3**).

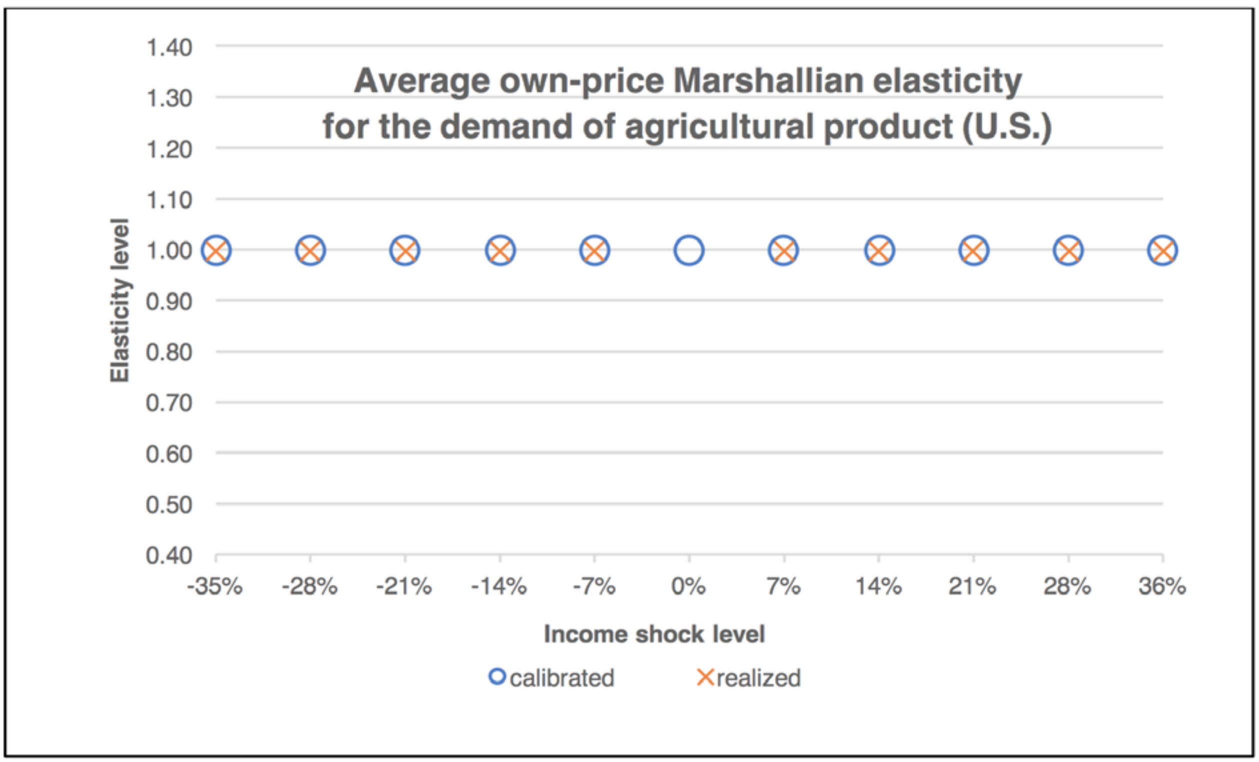

Figure 2. Average income elasticity for the agricultural product demand in the U.S.

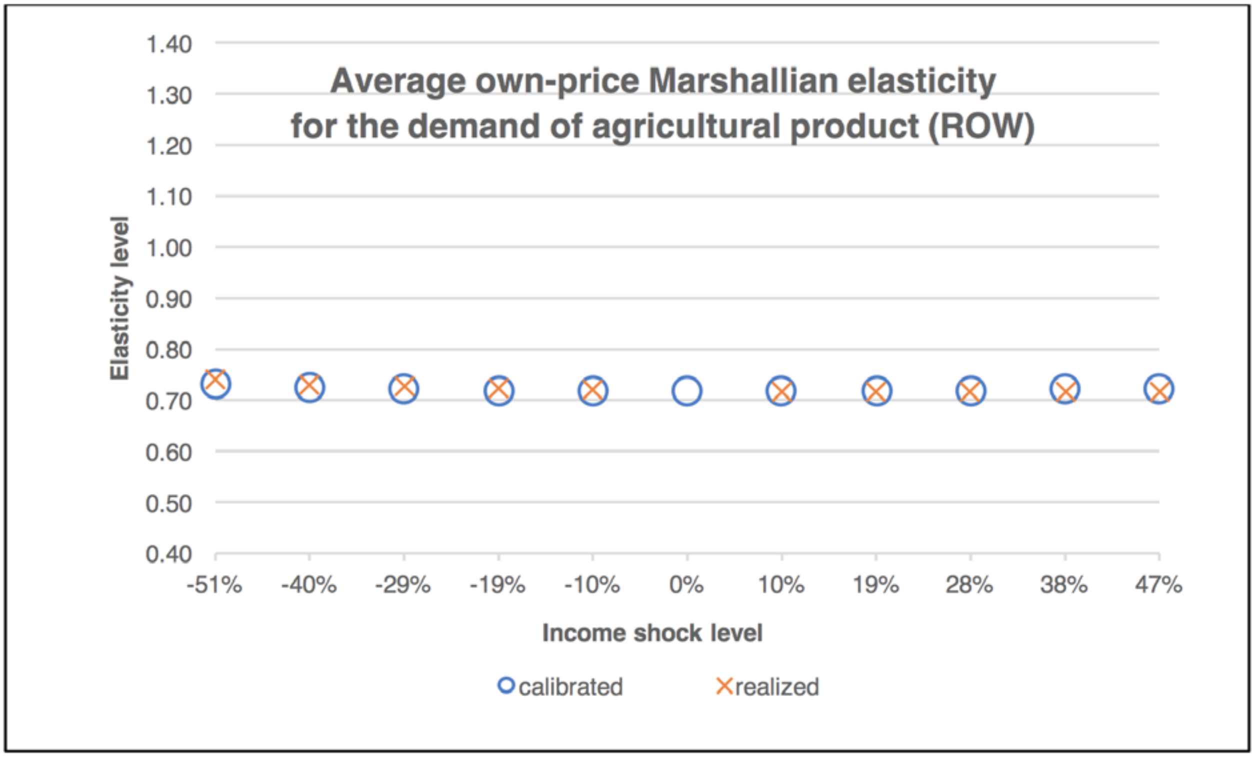

Figure 3. Average income elasticity for the agricultural product demand in the rest of world

4. Conclusion

This is the first paper to explore the circumstances under which the calibrated own-price and income elasticities of demand in a CDE demand system can be matched more accurately to their target levels. It finds that while the system is neither own-price nor income flexible, the elasticity match improves with lower sectorial expenditure shares (or a higher sectorial resolution), lower target own-price demand elasticities, and higher target income demand elasticities. In any case, to understand the extent to which the elasticity targets are correctly represented in a CGE model, it is crucial to check whether the target elasticities are valid (i.e., compatible to a NSD Slutsky matrix), and disclose how well the calibrated elasticities match their target counterparts. Without having these inspections, when the calibrated elasticities deviate from target levels, it will not be possible to determine if that is due to targeting elasticity levels that are invalid, or if the inflexibility of the demand system is indeed the cause of the mismatch.

In addition, using GTAP8inGAMS, the study also incorporates the CDE demand system into a global CGE model written in MPSGE, which has not been presented before. Furthermore, price and income shocks are imposed on this revised GTAP8inGAMS, and the model responses successfully replicate the calibrated elasticities of the CDE demand system. Future studies may examine if other CGE applications with the CDE demand can produce results consistent to the calibrated elasticities, or they may investigate the flexibility and calibration performance of other demand systems. These issues are rarely studied, but are essential for reasons discussed in this research.

Acknowledgments

The author gratefully acknowledges the financial support for this work provided by the MIT Joint Program on the Science and Policy of Global Change through a consortium of industrial and foundation sponsors and Federal awards, including the U.S. Department of Energy, Office of Science under DEFG02-94ER61937 and the U.S. Environmental Protection Agency under XA83600001-1. For a complete list of sponsors and the U.S. government funding sources, please visit http://globalchange.mit.edu/sponsors/all. The discussions with Tom Rutherford about the regularity and flexibility of a demand system are instrumental to this study. Also, the author is thankful for comments from Tom Hertel, Ching-Cheng Chang, three anonymous reviewers, participants of the MIT EPPA meeting, the 18th GTAP Conference in Melbourne, Australia, and the 2015 Taiwan Economic Association annual conference. Edits and suggestions from Jamie Bartholomay are highly appreciated. All remaining errors are my own.

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Appendix A: The CDE Calibration Program[7]

cdecalib.gms

```
$title Calibrate a CDE Demand System using GTAP data
$ontext
 Consider the following implicit function that defines the utility u:
 G(z,u) = sum(i,BETA(i)*u**(e(i)*(1-ALPHA(i)))*z(i)**(1-ALPHA(i))) = 1 where z(i) = p0(i)/c0
 p0 = benchmark price index, and c0 is the benchmark expenditure level

 Allen partial elasticities of substitution:
 sigma(i,j) = ALPHA(i)+ALPHA(j)-sum(k,theta(k)*ALPHA(k))-delta(i,j)*ALPHA(i)/theta(i);

 Income elasticities:
 eta(i) = (sum(k,theta(k)*e(k))**(-1))*(e(i)*(1-ALPHA(i))+sum(k,theta(k)*e(k)*ALPHA(k)))
        + (ALPHA(i)-sum(k,theta(k)*ALPHA(k)));

 The upper and lower bounds for ALPHA, e, and BETA come from the CDE regulatory conditions:
 substitution coefficient ALPHA(i) in (0,1) for all i (or ALPHA > 1 for all i)
 expansion parameter e(i) >= 0
 scale parameter BETA(i) >= 0
 References: Takeda (2012); Hertel et al. (1997); Hanoch (1975)
 Equations must be declared with static sets, but they can be assigned with dynamic sets.
YHC: 11/26/2014
$offtext

$if not set ds $set ds 2r4s1f
$if not set datadir $set datadir .\input\
$if not set wt $set wt 0
$include gtap8data_old

set  info          Information about this calibration /
     ds            "%ds%",
     datadir       "%datadir%",
     workdir       "%gams.workdir%"
     date          "%system.date%"
     time          "%system.time%" /;

alias(i,j,k);

set rr(r) dynamic subset of r;
rr(r) = no;

parameters
z(i,r)             normalized price
theta(i,r)         value share in final demand
vafm(i,r)          Aggregate final demand,
delta(i,j,r)       diagonal-one off-diagonal-zero
sigma(i,j,r)       Allen partial elasticity of substitution
epsilon_(i,r)      targeted own-price elasticity of demand
eta_(i,r)          targeted income elasticity of demand
p0(i,r)            benchmark price index
q0(i,r)            benchmark consumption level
c0(r)              expenditure level
mc0(r)             marginal cost when u is one
weight(i,r)        weight for the square distance
beta(i,r)          scale coefficient

;

vafm(i,r)          = vdfm(i,"c",r)*(1+rtfd0(i,"c",r))+vifm(i,"c",r)*(1+rtfi0(i,"c",r));
theta(i,r)         = vafm(i,r) / (vom("c",r)*(1-rto("c",r)));
abort$sum(r, round(abs(1-sum(i,theta(i,r))),5)) "Shares do not add up.";

epsilon_(i,r)      = epsilon(i,r);
eta_(i,r)          = eta(i,r);
p0(i,r)            = 1;
q0(i,r)            = theta(i,r)/p0(i,r);
c0(r)              = sum(i,p0(i,r)*q0(i,r));
delta(i,j,r)       = 0;
delta(i,j,r)$sameas(i,j) = 1;
weight(i,r)        = theta(i,r)$(%wt% eq 0) + (1/card(j))$(%wt% ne 0);

*    Finish reading data
*    -------------------------------------------------------
```

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7 This GAMS program implements the three-step procedure for calibrating the CDE system. To run it, one needs: 1) the GTAP 8 data in the gdx format (created by GTAP8inGAMS) with desired resolutions for regions, sectors, and primary factors; 2) the subroutine "gtap8data.gms," which is also included in GTAP8inGAMS, that reads data needed in the calibration program; 3) to type "gams cdecalib" under the DOS command prompt—this will use the default database "2r4s1f.gdx". The environment variable "ds" can be used to overwrite the default database setting.

cdecalib.gms

```
variables
ALPHA(i,r)          substitution coefficient
V(i,r)              own-price elasticity of demand
E(i,r)              expansion coefficient
ETAV(i,r)           income elasticity of demand
*BETA(i,r)          scale coefficient
OBJONE              objective value for own-price elasticity calibration
OBJTWO              objective value for income elasticity calibration
OBJTHR              objective value for the dummy
OBJFOR              objective value for the dummy
U(r)                utility
;

*    The equation "e_engel" deals with the case where eta from data doesn't satisfly the Engel aggregation

equations
e_v(i,r)            for v
e_eta(i,r)          for ETAV
e_objone            for OBJONE
e_objtwo            for OBJTWO
e_objthr            for OBJTHR
e_objfor            for OBJFOR
e_engel(r)          Engel aggregation
e_etaside(i,r)      ensure eta & eta_ lies on the same side of one
e_exp(r)            expenditure function
e_dfn(i,r)          compensated demand
;

*    Step 1: Calibrating to the own-price elasticity of demand

e_v(i,rr) ..
V(i,rr)$theta(i,rr) =e= theta(i,rr)*(2*ALPHA(i,rr)-sum(k,theta(k,rr)*ALPHA(k,rr)))-ALPHA(i,rr);

e_objone ..
OBJONE =e= sum((i,rr),weight(i,rr)*(V(i,rr)-epsilon_(i,rr))*(V(i,rr)-epsilon_(i,rr)));
*OBJONE =e= -sum((i,rr),V(i,rr)*(log(V(i,rr)/epsilon_(i,rr))-1));

model demandelas / e_v, e_objone /;

loop(r,
rr(r)               = yes;
ALPHA.L(i,rr)       = 0.5;
ALPHA.UP(i,rr)      = 0.99999;
ALPHA.LO(i,rr)      = 0.00001;
*ALPHA.LO(i,r)      = 1.00001;


V.L(i,rr)           = epsilon_(i,rr);
OBJONE.L            = 0;
solve demandelas using nlp minimizing OBJONE;
sigma(i,j,r)$theta(i,r)= ALPHA.L(i,r)+ALPHA.L(j,r)-sum(k,theta(k,r)*ALPHA.L(k,r))-delta(i,j,r)*ALPHA.L(i,r)/theta(i,r);
rr(r)               = no;
);

*    Step 2: Calibrating the income elasticity of demand

e_eta(i,rr) ..
ETAV(i,rr) =e= (1/sum(k,theta(k,rr)*E(k,rr)))*(E(i,rr)*(1-ALPHA.L(i,rr))+sum(k,theta(k,rr)*E(k,rr)*ALPHA.L(k,rr)))
      + (ALPHA.L(i,rr)-sum(k,theta(k,rr)*ALPHA.L(k,rr)));

e_objtwo ..
OBJTWO =e= sum((i,rr),weight(i,rr)*(ETAV(i,rr)-eta_(i,rr))*(ETAV(i,rr)-eta_(i,rr)));

e_engel(rr) ..
sum(i,theta(i,rr)*ETAV(i,rr)) =e= 1;

e_etaside(i,rr) ..
(ETAV(i,rr)-1)*(eta_(i,rr)-1) =g= 0;

model incomeelas /e_objtwo, e_engel, e_eta, e_etaside/;
```

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cdecalib.gms

```
model incomeelas /e_objtwo, e_engel, e_eta, e_etaside/;

loop(r,
rr(r)       = yes;
E.LO(i,rr)   = 0.0;
E.L(i,rr)    = 1;
ETAV.L(i,rr)  = eta_(i,r);

OBJTWO.L     = 0;
solve incomeelas using nlp minimizing OBJTWO;
rr(r)       = no;
);

*    Step 3: Calibrating the scale coefficient BETA holding the utility level equals one

beta(i,r) = (q0(i,r)/(1-ALPHA.L(i,r)))/sum(j,q0(j,r)/(1-ALPHA.L(j,r)));
U.FX(r)  = 1;

parameter  epsilonv00(i,r)  EPSILONV solved by the CDE calibration routine,
       etav00(i,r)     ETAV solved by the CDE calibration routine
       alpha00(i,r)    ALPHA solved by the CDE calibration routine
       e00(i,r)        E solved by the CDE calibration routine
       u00(r)          U solved by the CDE calibration routine
       beta00(i,r)     beta solved by the CDE calibration routine
       mc00(r)         Marginal cost
;

epsilonv00(i,r) = V.L(i,r);
etav00(i,r)    = ETAV.L(i,r);
alpha00(i,r)   = ALPHA.L(i,r);
e00(i,r)       = E.L(i,r);
u00(r)         = U.L(r);
beta00(i,r)    = beta(i,r);


mc00(r) =

 c0(r)*sum(i,beta(i,r)*E.L(i,r)*(1-ALPHA.L(i,r))*(U.L(r)**(E.L(i,r)*(1-ALPHA.L(i,r))-1))*(p0(i,r)/c0(r))**(1-ALPHA.L(i,r)))
    /sum(i,beta(i,r)*(1-ALPHA.L(i,r))*(U.L(r)**(E.L(i,r)*(1-ALPHA.L(i,r))))*(p0(i,r)/c0(r))**(1-ALPHA.L(i,r)));

parameter data model output;
data(i,r,"theta")      = theta(i,r);
data(i,r,"epsilon")    = epsilon_(i,r);
data(i,r,"epsilonv00") = epsilonv00(i,r);
data(i,r,"eta")        = eta_(i,r);
data(i,r,"etav")       = ETAV.L(i,r);
data(i,r,"alpha")      = ALPHA.L(i,r);
data(i,r,"e")          = E.L(i,r);
data(i,r,"beta")       = beta(i,r);
data("mc",r,"mc")      = mc00(r);
data(i,r,"weight")     = weight(i,r);

execute_unload ".\output\cdecalib_%ds%_wt=%wt%.gdx";
*execute_unload ".\output\cdecalib_%ds%_lnobj.gdx";
```

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Appendix B: The CGE Model with CDE Demand for GTAP8inGAMS[8]

```
                                              mrtmge_cde.gms
$title      Read GTAP8 Basedata and Replicate the Benchmark in MPSGE
* To run the model, type, for example: gams mrtmge_cde --start=0.1 --end=20 --step=0.1

* The following pre-assignment for ds will be used in a $gdxin command in gtap8data.gms
$if not set ds $set ds 2r4s1f

$if not set wt $set wt 0

* Sets, parameters declarations and assignments are done in gtap8data.gms (YHC: 20120614)
$include ..\build\gtap8data

set c(g) private consumption /c/;
set e(g) exogenous consumption /g, i/;

parameters
esub(g)              Top-level elasticity in demand /C 1/
vcm(i,c,r)           Tax included Armington good i for private consumption,
data(*,*,*)          Output from cdecalib,
cde                  CDE calibration,
chkd(i,r)            Check final expenditure D;

* Aggregate final demand (Armington good)
vcm(i,c,r) = vdfm(i,c,r)*(1+rtfd0(i,c,r))+vifm(i,c,r)*(1+rtfi0(i,c,r));

* Read the CDE coefficients
execute_load ".\input\cdecalib_%ds%_wt=%wt%.gdx" data = data;
cde(i,r,"alpha")                                   = data(i,r,"alpha");
cde(i,r,"e")                                       = data(i,r,"e");
cde(i,r,"beta")                                    = data(i,r,"beta");
cde("utility",r,"u")                               = 1;
cde("mc",r,"mc")                                   = data("mc",r,"mc");
cde(i,r,"alpha")$(cde(i,r,"alpha") eq eps)         = 0;
cde(i,r,"e")$(cde(i,r,"e") eq eps)                 = 0;
cde(i,r,"beta")$(cde(i,r,"beta") eq eps)           = 0;
cde("utility",r,"u")$(cde("utility",r,"u") eq eps) = 0;
cde("mc",r,"mc")$(cde("mc",r,"mc") eq eps)         = 0;

$ontext
$model:gtap8

$sectors:
        y(g,r)$(not c(g) and vom(g,r))   ! Supply
        m(i,r)$vim(i,r)                  ! Imports
        yt(j)$vtw(j)                     ! Transportation services
        ft(f,r)$(sf(f) and evom(f,r))    ! Specific factor transformation
        yc(i,c,r)$vcm(i,c,r)             ! Private consumption by commodity

$commodities:
        p(g,r)$vom(g,r)                  ! Domestic output price
        pm(j,r)$vim(j,r)                 ! Import price
        pt(j)$vtw(j)                     ! Transportation services
        pf(f,r)$evom(f,r)                ! Primary factors rent
        ps(f,g,r)$(sf(f) and vfm(f,g,r))  ! Sector-specific primary factors
        pc(i,c,r)$vcm(i,c,r)             ! Private consumption price

$consumers:
        ra(r)                            ! Representative agent

$auxiliary:
        TC(r)                            ! Expenditure for the CDE system
        U(r)                             ! Activity level of Utility
        D(i,r)$vcm(i,"c",r)              ! Activity level of final consumption

* Sectoral output
$prod:y(j,r)$vom(j,r)          s:esub(j)     i.tl:esubd(i)  va:esubva(j)
        o:p(j,r)     q:vom(j,r)     a:ra(r)  t:rto(j,r)
        i:p(i,r)     q:vdfm(i,j,r)         p:(1+rtfd0(i,j,r))    i.tl:  a:ra(r)  t:rtfd(i,j,r)
        i:pm(i,r)    q:vifm(i,j,r)         p:(1+rtfi0(i,j,r))    i.tl:  a:ra(r)  t:rtfi(i,j,r)
        i:ps(sf,j,r) q:vfm(sf,j,r)         p:(1+rtf0(sf,j,r))    va:    a:ra(r)  t:rtf(sf,j,r)
        i:pf(mf,r)   q:vfm(mf,j,r)         p:(1+rtf0(mf,j,r))    va:    a:ra(r)  t:rtf(mf,j,r)

                                              Page 1
```

8 To run this MPSGE program "mrtmge_cde.gms," one needs to 1) place it inside the subdirectory "model" of GTAP8inGAMS; 2) set either price shock or income shock within the loop; 3) set the output file name that distinguishes price shock from income shock; and 4) type, for example, "gams mrtmge_cde --start=0.1 --end=20 --step=0.1" under the DOS command prompt. With the default setting, this will produce 20 different price shocks for the agricultural product—the first shock will be created by multiplying both vdfm("agri",c,"usa") and vifm("agri",c,"usa") by 0.1, and for each following shock, the multiplicand increases by 0.1 compared to that in the previous shock.

mrtmge_cde.gms

```
* Government consumption and investment (exogenous consumption)
$prod:y(e,r)$vom(e,r)       s:esub(e)    i.tl:esubd(i)
        o:p(e,r)   q:vom(e,r)          a:ra(r)  t:rto(e,r)
        i:p(i,r)   q:vdfm(i,e,r)       p:(1+rtfd0(i,e,r))    i.tl:  a:ra(r)  t:rtfd(i,e,r)
        i:pm(i,r)  q:vifm(i,e,r)       p:(1+rtfi0(i,e,r))    i.tl:  a:ra(r)  t:rtfi(i,e,r)

* Private consumption: new
* Level 1: Armington good of commodity i
$prod:yc(i,c,r)$vcm(i,c,r) s:esubd(i)
        o:pc(i,c,r) q:vcm(i,c,r)       a:ra(r)  t:rto(c,r)
        i:p(i,r)    q:vdfm(i,c,r)      p:(1+rtfd0(i,c,r))    a:ra(r)  t:rtfd(i,c,r)
        i:pm(i,r)   q:vifm(i,c,r)      p:(1+rtfi0(i,c,r))    a:ra(r)  t:rtfi(i,c,r)

* Level 2: Aggregate various goods to a single consumption good c.  This is where we need to work on for CDE.
* Let's temporarily remove the declaration of y(c,r), and move the sources and sinks in this block to demand block.
* This strategy is similar to linking the top-down and bottom-up.
* Now this is moved to the representative agent block.

$prod:yt(j)$vtw(j)    s:1
        o:pt(j)     q:vtw(j)
        i:p(j,r)    q:vst(j,r)

$prod:m(i,r)$vim(i,r)          s:esubm(i)    s.tl:0
        o:pm(i,r)  q:vim(i,r)
        i:p(i,s)      q:vxmd(i,s,r)         p:pvxmd(i,s,r)        s.tl:  a:ra(s)  t:(-rtxs(i,s,r))  a:ra(r)  t:(rtms(i,s,r)*(1-rtxs(i,s,r)))
        i:pt(j)#(s) q:vtwr(j,i,s,r)         p:pvtwr(i,s,r)        s.tl:  a:ra(r)  t:rtms(i,s,r)

$prod:ft(sf,r)$evom(sf,r)  t:etrae(sf)
        o:ps(sf,j,r)q:vfm(sf,j,r)
        i:pf(sf,r)              q:evom(sf,r)

$demand:ra(r)
        d:p("c",r)              q:vom("c",r)
        e:p("c",rnum)           q:vb(r)
        e:p("g",r)              q:(-vom("g",r))
        e:p("i",r)              q:(-vom("i",r))
        e:pf(f,r)               q:evom(f,r)
        e:p(c,r)                q:vom(c,r)              r:U(r)
        e:pc(i,c,r)             q:(-vcm(i,c,r))         r:D(i,r)

$constraint:TC(r)
    sum(i,cde(i,r,"beta")*(cde("utility",r,"u")*U(r))**(cde(i,r,"e")*(1-cde(i,r,"alpha")))*
    (PC(i,"c",r)/TC(r))**(1-cde(i,r,"alpha"))) =e= 1;

$constraint:U(r)
    TC(r)*sum(i,cde(i,r,"beta")*cde(i,r,"e")*(1-cde(i,r,"alpha"))*(U(r)**(cde(i,r,"e")*(1-cde(i,r,"alpha"))-1))*(PC(i,"c",r)/TC(r))**(1-cde(i,r,"alpha")))
     =e=
data("mc",r,"mc")*P("c",r)*sum(i,cde(i,r,"beta")*(1-cde(i,r,"alpha"))*(U(r)**(cde(i,r,"e")*(1-cde(i,r,"alpha"))))*(PC(i,"c",r)/TC(r))**(1-cde(i,r,"alpha")));

$constraint:D(i,r)$(vcm(i,"c",r))
    vcm(i,"c",r)/vom("c",r)*D(i,r)*sum(j,cde(j,r,"beta")*(U(r)**((1-cde(j,r,"alpha"))*cde(j,r,"e")))*(1-cde(j,r,"alpha"))*(PC(j,"c",r)/TC(r))**(1-cde(j,r,"alpha")))

     =e= (cde(i,r,"beta")*(U(r)**((1-cde(i,r,"alpha"))*cde(i,r,"e")))*(1-cde(i,r,"alpha"))*(pc(i,"c",r)/TC(r))**(-cde(i,r,"alpha")));
$offtext
$sysinclude mpsgeset gtap8

TC.L(r)             = 1;
TC.LO(r)            = 0.000001;
U.L(r)              = 1;
U.LO(r)             = 0.000001;
D.L(i,r)            = 1;
D.LO(i,r)           = 0.000001;
PF.FX("primary","usa") = 1;

gtap8.workspace     = 128;
gtap8.iterlim       = 0;
$include gtap8.gen
solve gtap8 using mcp;

chkd(i,r) = vcm(i,"c",r)/vom("c",r)*D.L(i,r)
     - (cde(i,r,"beta")*(U.L(r)**((1-cde(i,r,"alpha"))*cde(i,r,"e")))*(1-cde(i,r,"alpha"))*(PC.L(i,"c",r)/TC.L(r))**(-cde(i,r,"alpha")))
```

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mrtmge_cde.gms

```
/sum(j,cde(j,r,"beta")*(U.L(r)**((1-cde(j,r,"alpha"))*cde(j,r,"e")))*(1-cde(j,r,"alpha"))*PC.L(j,"c",r)**(1-cde(j,r,"alpha"))*TC.L(r)**cde(i,r,"alpha"));

execute_unload ".\output\mrtmge_cde_ref_ds=%ds%.gdx";

* The code below is for testing whether the model's realized elasticities equal the calibrated levels it is given to

$if not set step $set step 0
set x shock level /1*%end%/
parameters
step            step of the shock level,
start           initial shock coefficient,
vdfm0           vdfm value from GTAP,
vifm0           vifm value from GTAP,
evom0           evom value from GTAP,
pfx             realized PF with shock level x,
pcx             realized PC over PF with shock level x,
dx              realized D with shock level x,
theta_i         final consumption expenditure share,
eta_i           calibrated income demand point elasticity,
priexp          total private expenditure,
priexpi         total private expenditure index,
eta_i_a         calibrated average income demand elasticity,
sigma           calibrated AUES price demand elasticity (point elasticity),
delta(i,j,r)    diagonal-one off-diagonal-zero,
sigma_c         calibrated compensated price demand elasticity (point elasticity),
sigma_m         calibrated Marshallian price demand elasticity (point elasticity),
sigma_ma        calibrated Marshallian price demand elasticity (average elasticity),
dxn             realized D with shock level x net of prices & income effects,
sigma_mar       realized Marshallian price elasticity (average elasticity),
cds             change in d due to changes in other prices,
cdi             change in d due to change in income,
eqi             expected quantity due to pure income effect,
cqp             change in quantity due to change in own-price,
dxi             realized D with shock level x net of prices effects,
eta_i_ar        realized average income demand elasticity;

alias(i,k);

* Assign start and step in the command line using environment variables
start     = %start%;
step      = %step%;

* Read the shares and calibrated elasticities
theta_i(i,r,"theta")    = data(i,r,"theta");
eta_i(i,r,"etav")       = data(i,r,"etav");

* Store the original vdfm, vifm, and evom in GTAP
vdfm0(i,c,r)          = vdfm(i,c,r);
vifm0(i,c,r)          = vifm(i,c,r);
evom0(f,r)            = evom(f,r);

* Step 1: Calculate the Marshallian price demand elasticity (point elasticity)
delta(i,j,r)          = 0;
delta(i,j,r)$sameas(i,j) = 1;
sigma(i,j,r)          = cde(i,r,"alpha")+cde(j,r,"alpha")-sum(k, theta_i(k,r,"theta")*cde(k,r,"alpha"))
                        -delta(i,j,r)*cde(i,r,"alpha")/theta_i(i,r,"theta");
sigma_c(i,j,r)        = sigma(i,j,r)*theta_i(i,r,"theta");
sigma_m(i,j,r)        = sigma_c(i,j,r)-eta_i(i,r,"etav")*theta_i(i,r,"theta");

loop(x,

* Consumer's price shock:
vdfm("agri",c,"usa")          = vdfm0("agri",c,"usa")*(start+(ord(x)-1)*step);
vifm("agri",c,"usa")          = vifm0("agri",c,"usa")*(start+(ord(x)-1)*step);
*vdfm("agri",c,"row")         = vdfm0("agri",c,"row")*(start+(ord(x)-1)*step);
*vifm("agri",c,"row")         = vifm0("agri",c,"row")*(start+(ord(x)-1)*step);

* Endowment shock:
*evom(f,"usa")                = evom0(f,"usa")*(start+(ord(x)-1)*step);
*evom(f,"row")                = evom0(f,"row")*(start+(ord(x)-1)*step);
```

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mrtmge_cde.gms

```
* Avoid raising 0 by a negative number in the third auxiliary equation
PC.LO(i,"c",r) = 0.000001;

gtap8.iterlim = 50000;
$include gtap8.gen
solve gtap8 using mcp;

* Step 2: Within the loop, derive the calibrated average elasticities associated with the shock

** Marshallian price demand elasticity (average elasticity)
pfx(r,x)  = PF.L("primary",r);
pcx(i,r,x) = PC.L(i,"c",r)/pfx(r,x);
sigma_ma(i,j,r,x)$(pcx(j,r,x) ne 1) = (pcx(j,r,x)**sigma_m(i,j,r)-1)/(pcx(j,r,x)-1)
                    *(pcx(j,r,x)+1)/(pcx(j,r,x)**sigma_m(i,j,r)+1);
sigma_ma(i,j,r,x)$(pcx(j,r,x) eq 1) = sigma_m(i,j,r);

** Income demand elasticity (average elasticity)
priexp(r,x) = sum(i,pcx(i,r,x)*D.L(i,r)*vcm(i,"c",r));
priexpi(r,x) = priexp(r,x)/sum(i,vcm(i,"c",r));
eta_i_a(i,r,x)$(priexpi(r,x) ne 1) = (priexpi(r,x)**eta_i(i,r,"etav")-1)/(priexpi(r,x)-1)
                    *(priexpi(r,x)+1)/(priexpi(r,x)**eta_i(i,r,"etav")+1);
eta_i_a(i,r,x)$(priexpi(r,x) eq 1) = eta_i(i,r,"etav");

* Step 3-1: Calculate the substitution effect due to changes in PC-others|original income; after shock PC-own
dx(i,r,x)   = D.L(i,r);
cds(i,r,x)  = sum(j$(not sameas(i,j)),
(pcx(j,r,x)-1)/((pcx(j,r,x)+1)/2)*sigma_ma(i,j,r,x)*(1+sigma_ma(i,i,r,x)*(pcx(i,r,x)-1)/((1+pcx(i,r,x))/2)*((1+dx(i,r,x))/2)));

* Step 3-2: Calculate the income effect on top of changes in all PC
cdi(i,r,x)  = (priexpi(r,x)-1)/((priexpi(r,x)+1)/2)*eta_i_a(i,r,x)*(1+sigma_ma(i,i,r,x)*(pcx(i,r,x)-1)/((1+pcx(i,r,x))/2)+cds(i,r,x));

* Step 3-3: Calculate the adjusted demand net of cross-price effect and income effect
dxn(i,r,x)  = dx(i,r,x)-cds(i,r,x)-cdi(i,r,x);
sigma_mar(i,j,r,x)$(pcx(j,r,x) ne 1) = (dxn(i,r,x)-1)/((dxn(i,r,x)+1)/2) / ((pcx(j,r,x)-1)/((pcx(j,r,x)+1)/2));

* Step 3-4: Calculate expected quantity level due to pure income effect
eqi(i,r,x)  = (priexpi(r,x))**eta_i(i,r,"etav");

* Step 3-5: Based on the expected quantity derived from pure income effect, calculate the quantity changes due to changes in prices
cqp(i,r,x)  = sum(j, (pcx(j,r,x)-1)/((pcx(j,r,x)+1)/2)*sigma_ma(i,j,r,x)*eqi(i,r,x));

* Step 3-6: Substract quantity change due to price effect from the observed quantity
dxi(i,r,x)  = dx(i,r,x) - cqp(i,r,x);

* Step 3-7: Calculate the realized income demand elasticity
eta_i_ar(i,r,x) = (dxi(i,r,x)-1)/((dxi(i,r,x)+1)/2)/((priexpi(r,x)-1)/((priexpi(r,x)+1)/2));

execute_unload ".\output\mrtmge_cde_policy_ds=%ds%_priceshock=%step%.gdx";
*execute_unload ".\output\mrtmge_cde_policy_ds=%ds%_incomeshock=%step%.gdx";

);
```

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Appendix C: The Program Checking if Elasticity Targets are Valid

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```
% Read EXCEL input: share; eps_target; eps_calib; eta_target; eta_calib
data = xlsread('.\input\elastheta.xlsx','4x4','B3:F6');

%{
data in the worksheet "4x4"

sector share  eps_target eps_calib  eta_target eta_calib
s01    0.1178 -0.4294    -0.4657    0.7300     0.8442
s02    0.2479 -0.6650    -0.7201    0.9997     1.0000
s03    0.3955 -0.7800    -0.5767    1.0543     1.0289
s04    0.2388 -0.7424    -0.7445    1.0435     1.0289
%}

% Declare dimension
n = 4;

% Check Engel's aggregation (variable engel = 1 must hold)
eta_target = data(1:n, 4:4);
theta = data(1:n, 1:1);
engel = theta'*eta_target;

% Create a diagonal matrix with diagonal terms being the own-price AUES elasticities
eps_target = data(1:n, 2:2);
theta_diag = diag(theta);
aues_diag = diag(inv(theta_diag)*eps_target);

% Initialize the determinants for checking ND (sa stores values of various determinants)
sa = zeros(n,1);
for i = 1:n-1
  sa(i) = (-1)^(i+1);
end

while sa(1)>0|sa(2)<0|sa(3)>0|abs(sa(4))>0.00000001

% Empty aues from the previous run
aues_off = zeros(n,n);

% For each row create random variables no larger than the |diagonal term|/n
for i = 1:n-3
  offi = (-1+2*rand)*abs(aues_diag(i,i))/n;
  for j = i+1:n-1
      aues_off(i,j) = offi;
      aues_off(j,i) = aues_off(i,j);
  end
end

aues = aues_diag + aues_off;

% Create the "A" (LHS coefficient) matrix for solving the unknowns
A = zeros(n,n);
```

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```
for i = 1:n-1
   A(i,i) = theta(n,1);
   A(n,i) = theta(i,1);
end
A(n-2,n) = theta(n-1,1);
A(n-1,n) = theta(n-2,1);

% This incomplete aues matrix is suitable for finding "C" (RHS coefficient) matrix
C = -aues*theta;

% The unknowns are in "B" and are solved by A*B = C
B = inv(A)*C;

% Assign "B" to uknowns in aues, and now all aues unknows are found
for i = 1:n-1
   aues(i,n) = B(i,1);
end
aues(n-2,n-1) = B(n,1);

% Assign the solved AUES unknows (i,j) to their corresponding (j,i) elements
for i = 1:n
   for j= 1:n
aues(j,i) = aues(i,j);
   end
end

% Check Cournot aggregation
cournot = aues*theta;

% Check NSD
for i = 1:n
sa(i) = det(aues(1:i, 1:i)/10);
end

end
```

Appendix D: Calibration Details of the CDE System

	θ	α	ϵ_{target}	$\epsilon_{calibrated}$	e	η_{target}	$\eta_{calibrated}$
1r3s2f							
s01	0.11779	0.69132	-0.42935	-0.64196	1.00000	0.72997	0.99993
s02	0.24791	0.99999	-0.66503	-0.74307	0.00000	0.99974	1.00000
s03	0.63430	0.99999	-0.76584	-0.34264	1.39128	1.05025	1.00001
1r4s2f							
s01	0.11779	0.46593	-0.42935	-0.46570	1.69852	0.72997	0.84424
s02	0.24791	0.97108	-0.66503	-0.72013	0.00000	0.99974	1.00000
s03	0.23876	0.99999	-0.74242	-0.74450	0.00000	1.04350	1.02891
s04		0.99999	-0.77997	-0.57674	6.05841	1.05432	1.02894
1r5s2f							
s01	0.03432	0.25519	-0.20559	-0.26903	0.34455	0.55041	0.55041
s02	0.11603	0.64030	-0.52076	-0.59769	0.37632	0.81865	0.81865
s03	0.10413	0.81460	-0.66544	-0.74006	0.81022	1.00727	1.00727
s04	0.28309	0.99999	-0.75108	-0.69238	1.03185	1.05134	1.04791
s05	0.46242	0.99999	-0.76079	-0.49752	1.30986	1.04581	1.04791
1r8s2f							
s01	0.03352	0.21620	-0.19417	-0.23118	1.77964	0.53872	0.53872
s02	0.02279	0.52803	-0.48782	-0.52400	2.53677	0.81197	0.81197
s03	0.09404	0.60345	-0.53011	-0.57264	2.17642	0.82213	0.82213
s04	0.09783	0.77401	-0.66566	-0.70859	4.08792	1.00456	1.00456
s05	0.04364	0.79038	-0.72192	-0.75976	4.69953	1.03290	1.03289
s06	0.07051	0.78613	-0.69707	-0.73727	4.80226	1.03684	1.03683
s07	0.24214	0.99999	-0.74111	-0.72864	0.99992	1.04339	1.05016
s08	0.39553	0.99999	-0.77997	-0.55674	9.13649	1.05432	1.05018
1r16s2f							
s01	0.02937	0.15660	-0.16693	-0.17214	0.28241	0.48744	0.48744
s02	0.00415	0.39182	-0.38712	-0.39206	0.63398	0.90206	0.90206
s03	0.00092	0.67054	-0.66517	-0.67008	0.82138	1.04085	1.04085
s04	0.02187	0.48840	-0.48033	-0.48546	0.43837	0.80230	0.80230
s05	0.01917	0.42077	-0.41568	-0.42078	0.38537	0.73339	0.73339
s06	0.07487	0.59054	-0.55941	-0.56517	0.42817	0.84486	0.84486
s07	0.03095	0.65777	-0.63789	-0.64312	0.63971	0.96517	0.96517
s08	0.06687	0.72478	-0.67851	-0.68417	0.76744	1.02279	1.02279
s09	0.00330	0.65472	-0.64824	-0.65318	0.84878	1.05236	1.05236
s10	0.04035	0.76068	-0.72795	-0.73328	0.78856	1.03131	1.03131
s11	0.04451	0.74928	-0.71468	-0.72006	0.80920	1.03715	1.03715
s12	0.02600	0.68584	-0.66691	-0.67208	0.80951	1.03631	1.03631
s13	0.20777	0.99999	-0.75135	-0.75945	0.89823	1.05178	1.04092
s14	0.03437	0.70394	-0.67923	-0.68450	0.68774	0.99270	0.99270
s15	0.13072	0.93955	-0.79738	-0.80402	1.60728	1.09873	1.09873
s16	0.26481	0.99999	-0.77138	-0.69341	0.67100	1.03240	1.04092

	θ	α	ϵ_{target}	$\epsilon_{calibrated}$	e	η_{target}	$\eta_{calibrated}$
1r29s2f							
s01	0.00143	0.09267	-0.09357	-0.09357	0.42740	0.33991	0.33991
s02	0.01600	0.10522	-0.11479	-0.11479	0.40249	0.33825	0.33825
s03	0.00069	0.09841	-0.09883	-0.09883	0.62831	0.43748	0.43748
s04	0.00290	0.13154	-0.13312	-0.13312	0.47413	0.39110	0.39110
s05	0.00520	0.30895	-0.30995	-0.30995	1.33837	0.83221	0.83221
s06	0.00315	0.27425	-0.27506	-0.27506	1.39632	0.84299	0.84299
s07	0.00398	0.38575	-0.38590	-0.38590	1.46331	0.89563	0.89563
s08	0.00017	0.41754	-0.41753	-0.41753	1.97351	1.05623	1.05623
s09	0.00092	0.66584	-0.66536	-0.66536	1.90474	1.04089	1.04089
s10	0.01817	0.51877	-0.51461	-0.51461	1.00120	0.81432	0.81432
s11	0.01690	0.46672	-0.46461	-0.46461	1.05382	0.80348	0.80348
s12	0.00597	0.21068	-0.21299	-0.21299	0.69651	0.54116	0.54116
s13	0.06527	0.57659	-0.55410	-0.55410	0.93506	0.82800	0.82800
s14	0.02982	0.64004	-0.62598	-0.62598	1.45830	0.95776	0.95776
s15	0.01073	0.63809	-0.63307	-0.63307	1.58403	0.98065	0.98065
s16	0.03895	0.69602	-0.67330	-0.67330	1.68759	1.00760	1.00760
s17	0.03068	0.70071	-0.68252	-0.68252	1.92233	1.04433	1.04433
s18	0.00054	0.64150	-0.64124	-0.64124	2.09503	1.07543	1.07543
s19	0.03579	0.76118	-0.73564	-0.73564	1.84873	1.03603	1.03603
s20	0.01658	0.69629	-0.68660	-0.68660	1.81666	1.02778	1.02778
s21	0.03249	0.74450	-0.72240	-0.72240	1.85417	1.03591	1.03591
s22	0.02171	0.67285	-0.66119	-0.66119	1.88490	1.03770	1.03770
s23	0.00765	0.67912	-0.67491	-0.67491	1.85026	1.03219	1.03219
s24	0.20439	0.99418	-0.75305	-0.75305	3.31501	1.05204	1.05204
s25	0.00855	0.63947	-0.63545	-0.63545	1.67133	0.99708	0.99708
s26	0.06077	0.80464	-0.75598	-0.75598	2.10861	1.06429	1.06429
s27	0.09577	0.88842	-0.79570	-0.79570	2.70728	1.09161	1.09161
s28	0.17177	0.96520	-0.77251	-0.77251	1.01877	1.03136	1.03136
s29	0.09304	0.85273	-0.76929	-0.76929	1.76577	1.03432	1.03432
s01	0.00681	0.65433	-0.65067	-0.65067	0.30974	0.99476	0.99476
s02	0.02640	0.56879	-0.55911	-0.55911	0.18143	0.83196	0.83196
s03	0.00017	0.09099	-0.09111	-0.09109	0.09240	0.36960	0.36960
s04	0.02582	0.71052	-0.69373	-0.69373	0.29289	0.99124	0.99124
s05	0.00775	0.52036	-0.51827	-0.51827	0.18304	0.80914	0.80914
s06	0.00337	0.65002	-0.64823	-0.64823	0.35438	1.03589	1.03589
s07	0.00017	0.41702	-0.41692	-0.41701	0.37516	1.05659	1.05659
s08	0.02792	0.70352	-0.68575	-0.68575	0.36117	1.04398	1.04398
s09	0.00078	0.27528	-0.27545	-0.27545	0.28216	0.87675	0.87675
s10	0.09304	0.85704	-0.76927	-0.76927	0.32303	1.03432	1.03432
s11	0.01203	0.70142	-0.69382	-0.69382	0.35690	1.04050	1.04050
s12	0.01903	0.66669	-0.65599	-0.65599	0.35670	1.03880	1.03880
s13	0.00301	0.67887	-0.67710	-0.67711	0.36430	1.04588	1.04588

	θ	α	ϵ_{target}	$\epsilon_{calibrated}$	e	η_{target}	$\eta_{calibrated}$
s14	0.00105	0.48534	-0.48513	-0.48513	0.37597	1.05735	1.05735
s15	0.00293	0.35032	-0.35053	-0.35052	0.24937	0.83797	0.83797
s16	0.00081	0.68067	-0.68020	-0.68020	0.35220	1.03557	1.03557
s17	0.00273	0.69484	-0.69315	-0.69315	0.34460	1.03015	1.03015
s18	0.00152	0.11160	-0.11243	-0.11243	0.11440	0.43746	0.43746
s19	0.00026	0.60859	-0.60854	-0.60847	0.40425	1.08610	1.08610
s20	0.02590	0.77890	-0.75851	-0.75851	0.31258	1.01763	1.01763
s21	0.00739	0.60771	-0.60442	-0.60442	0.27705	0.95247	0.95247
s22	0.00334	0.69847	-0.69638	-0.69638	0.36011	1.04296	1.04296
s23	0.01320	0.51068	-0.50737	-0.50737	0.19535	0.82036	0.82036
s24	0.03278	0.76596	-0.74100	-0.74100	0.34384	1.03512	1.03512
s25	0.00028	0.67218	-0.67193	-0.67202	0.38674	1.06543	1.06543
s26	0.00276	0.65106	-0.64960	-0.64959	0.36712	1.04786	1.04786
s27	0.00442	0.31545	-0.31607	-0.31607	0.24823	0.82431	0.82431
s28	0.06985	0.87837	-0.80949	-0.80949	0.57418	1.11904	1.11904
s29	0.00239	0.06962	-0.07113	-0.07113	0.05907	0.27043	0.27043
s30	0.03887	0.56463	-0.55069	-0.55069	0.17756	0.82531	0.82531
s31	0.03495	0.83329	-0.80198	-0.80198	0.51806	1.11825	1.11825
s32	0.00000	0.54565	-0.54574	-0.54565	0.34258	1.01626	1.01626
s33	0.01632	0.73209	-0.72077	-0.72077	0.34887	1.03608	1.03608
s34	0.01617	0.73536	-0.72405	-0.72405	0.34810	1.03574	1.03574
s35	0.00012	0.56284	-0.56280	-0.56280	0.39380	1.07767	1.07767
s36	0.01042	0.51457	-0.51188	-0.51188	0.19225	0.81818	0.81818
s37	0.00052	0.10104	-0.10135	-0.10134	0.12667	0.45970	0.45970
s38	0.10513	0.88557	-0.78040	-0.78040	0.29993	1.03107	1.03107
s39	0.00456	0.67014	-0.66755	-0.66755	0.30611	0.99424	0.99424
s40	0.03252	0.62968	-0.61379	-0.61379	0.31758	0.99835	0.99835
s41	0.02743	0.66120	-0.64607	-0.64607	0.31057	0.99669	0.99669
s42	0.00350	0.10424	-0.10620	-0.10620	0.09453	0.38467	0.38467
s43	0.00029	0.12906	-0.12919	-0.12921	0.16415	0.56568	0.56568
s44	0.00052	0.41959	-0.41952	-0.41956	0.30542	0.94812	0.94812
s45	0.01153	0.74657	-0.73824	-0.73824	0.34381	1.03358	1.03358
s46	0.00296	0.27504	-0.27569	-0.27569	0.26158	0.83670	0.83670
s47	0.06663	0.81767	-0.76006	-0.76006	0.32874	1.03182	1.03182
s48	0.00248	0.36364	-0.36375	-0.36375	0.20224	0.76202	0.76202
s49	0.00960	0.59960	-0.59548	-0.59548	0.28551	0.95945	0.95945
s50	0.17186	0.98578	-0.77940	-0.77940	0.60041	1.06220	1.06220
s51	0.01448	0.10698	-0.11504	-0.11504	0.06992	0.32782	0.32782
s52	0.00370	0.31151	-0.31206	-0.31206	0.20503	0.74329	0.74329
s53	0.02022	0.65120	-0.64045	-0.64045	0.26986	0.95695	0.95695
s54	0.00115	0.08393	-0.08462	-0.08462	0.05940	0.28326	0.28326
s55	0.00019	0.26531	-0.26536	-0.26536	0.31734	0.94356	0.94356
s56	0.00174	0.57650	-0.57583	-0.57583	0.33148	1.00619	1.00619
s57	0.00428	0.69859	-0.69591	-0.69591	0.34315	1.02927	1.02927

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